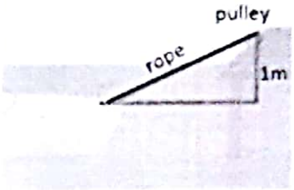
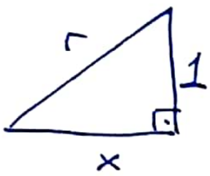
Name:	Department: Comp Eng	GRADE
Student No:	Calculus 1 Exam 1	
Signature:	Date: 06/11/2017	

`Demonstrate your solution steps clearly.  Do not use L'Hospital Rule in limit questions.`

1. (16 points) A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the dock when it is 8 m from the dock?



$\boxed{-\sqrt{65}/8}$



$x^2 + 1^2 = r^2 \Rightarrow 2x\frac{dx}{dt} = 2r\frac{dr}{dt}$

$x = 8 \Rightarrow r = \sqrt{65}$

$8\frac{dx}{dt} = \sqrt{65}\,(-1) \Rightarrow \frac{dx}{dt} = -\frac{\sqrt{65}}{8}$

2. (16 points) If $f(x) = e^{\ln(x^3)} + e^{\sin(2x-1)}$, then $f'(1/2) =$ $\boxed{11/4}$

$f(x) = x^3 + e^{\sin(2x-1)}$

$f'(x) = 3x^2 + e^{\sin(2x-1)}\cos(2x-1)\cdot 2$

$f'(1/2) = \frac{3}{4} + e^{\sin 0}\cos 0 \cdot 2 = \frac{3}{4} + 2$

3. (16 points) If $y = (x^2+2)^x$ then $y'(1) =$ $\boxed{3\ln 3 + 2}$

$y = e^{\ln(x^2+2)^x} = e^{x\ln(x^2+2)}$

$y' = e^{x\ln(x^2+2)}\cdot\left(1\cdot\ln(x^2+2) + x\cdot\frac{(2x+0)}{x^2+2}\right)$

$y'(1) = e^{\ln 3}\cdot\left(\ln 3 + \frac{2}{3}\right) = 3\left(\ln 3 + \frac{2}{3}\right)$

4. (16 points) $\lim_{x\to\infty}\left(x-\sqrt{x^2+7}\right) = \boxed{0}$

$$= \lim_{x\to\infty} \frac{(x-\sqrt{x^2+7})(x+\sqrt{x^2+7})}{x+\sqrt{x^2+7}} = \lim_{x\to\infty} \frac{-7}{x\left(1+\sqrt{1+7/x^2}\right)}$$

$$= \lim_{x\to\infty} \frac{-7/x}{1+\sqrt{1+7/x^2}} = \frac{0}{1+\sqrt{1+0}} = 0.$$

5. (16 points) Find the derivative of $f(x) = 3x^2-2x$ using the **limit definition of the derivative.** $\boxed{6x-2}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)^2-2(x+h)-3x^2+2x}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x^2}+6xh+3h^2-\cancel{2x}-2h-\cancel{3x^2}+\cancel{2x}}{h} = \lim_{h\to 0} \frac{6xh+3h^2-2h}{h}$$

$$= \lim_{h\to 0} 6x+3h-2 = 6x-2$$

6. (16 points) **Using the implicit differentiation** find the equation of the tangent line to the curve $x\sin(2y) = y\cos(2x)$ at the point $(x, y) = (\pi/4, \pi/2)$. $\boxed{y-\frac{\pi}{2} = 2(x-\pi/4)}$

$$\frac{d}{dx}(x\sin(2y)) = \frac{d}{dx}(y\cos(2x))$$

$$\sin 2y + x\cos(2y)\cdot 2y' = y'\cos 2x - y\sin 2x \cdot 2$$

$$\cancel{\sin\pi} + \frac{\pi}{4}\cos\pi \cdot 2y' = \cancel{y'\cos\pi} - \frac{\pi}{2}\sin\frac{\pi}{2}\cdot 2$$

$$\frac{-\pi}{4}\cdot 2y' = \frac{-\pi}{2}\cdot 2 \Rightarrow \boxed{y'=2} \Rightarrow$$

7. (8 points) Suppose f is a function such that $f'(x) \le 3$ for all $1 \le x \le 5$. **Using the Mean Value Theorem,** find the largest possible value of $f(5)-f(1)$. $\boxed{12}$

$$\frac{f(5)-f(1)}{5-1} = f'(c),\quad 1<c<5 \Rightarrow f(5)-f(1) = 4f'(c) \le 4\cdot 3$$